Multiphase Patterns in Forced Reaction-Diffusion Systems

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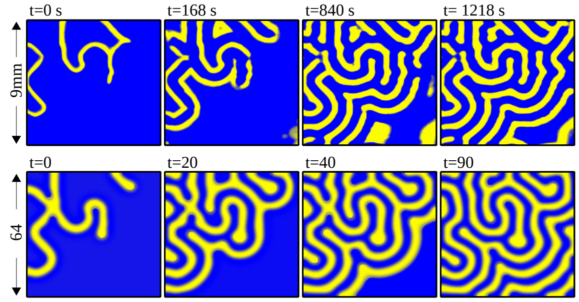
In 1665 Dutch Physicist Christiaan Huygens noticed that the two pendulum clocks hanging on his wall were swinging exactly together for as long as he cared to watch. After further experimenting he discovered that the common support on the wall coupled the two oscillators (pendula) and caused them to synchronize even though they had slightly different frequencies. Since then many scientists have studied synchronization of oscillators, including arrays of oscillators and the effect of external periodic forcing. External forcing, for example wiggling the clock support in Huygens' experiment, can lock the oscillators to multiples of the forcing period. This locking is seen in many natural systems such as the human sleep-wake cycle.

We are studying an oscillating chemical reaction system that can be synchronized by flashes from a light projector. The chemical reaction is prepared in a two-dimensional gel disk. This allows us to study not only the time dynamics of synchronization, as Huygens did, but also the spatial patterns. Spatial patterns form as regions with different phases; at any instant some clock pendulums are to the left and some to the right but all still keep the same time.

Remarkably, changing the frequency of the flashing light changes the pattern formed by the chemical oscillators even though the light is spatially uniform. For example, when flashed at near twice the unforced oscillating frequency, the system locks into one of two oscillation phases. We observe rotating spiral waves, standing-wave labyrinths, and other synchronized irregular patterns depending on the exact external forcing frequency.

Examples of a standing-wave labyrinthine patterns are shown in the figures below. The pattern is formed from different phases (yellow and blue) which are exactly π out of phase with each other.



The formation of a standing-wave labyrinthine pattern in experiment (top) and model (bottom).

Our modeling, using theory and numerical solution of partial differential equations, has allowed us to gain a new understanding of the patterns found in oscillatory systems. It also has produced new pattern solutions and provided insight into the dynamics of pattern formation. The figures show an example of a new mechanism we found for the formation of labyrinths in both the experiment and the numerical solution of our model.

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For further information and papers see `http://math.lanl.gov/~aric`

References

[1] A. Yochelis, A. Hagberg, E. Meron, A. L. Lin, and H. L. Swinney. Development of standing-wave labyrinthine patterns. *SIADS*, 1(2):236–247, 2002. LA-UR-01-5306.